

《群的上同调》

图书基本信息

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前言

This book is based on a course given at Cornell University and intended primarily for second-year graduate students. The purpose of the course was to introduce students who knew a little algebra and topology to a subject in which there is a very rich interplay between the two. Thus I take neither a purely algebraic nor a purely topological approach, but rather I use both algebraic and topological techniques as they seem appropriate. The first six chapters contain what I consider to be the basics of the subject. The remaining four chapters are somewhat more specialized and reflect my own research interests. For the most part, the only prerequisites for reading the book are the elements of algebra (groups, rings, and modules, including tensor products over non-commutative rings) and the elements of algebraic topology (fundamental group, covering spaces, simplicial and CW-complexes, and homology). There are, however, a few theorems, especially in the later chapters, whose proofs use slightly more topology (such as the Hurewicz theorem or Poincaré duality). The reader who does not have the required background in topology can simply take these theorems on faith. There are a number of exercises, some of which contain results which are referred to in the text. A few of the exercises are marked with an asterisk to warn the reader that they are more difficult than the others or that they require more background. I am very grateful to R. Bieri, J-P. Serre, U. Stambach, R. Strebél, and C. T. C. Wall for helpful comments on a preliminary version of this book.

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内容概要

《群的上同调》讲述了：This book is based on a course given at Cornell University and intended primarily for second-year graduate students. The purpose of the course was to introduce students who knew a little algebra and topology to a subject in which there is a very rich interplay between the two. Thus I take neither a purely algebraic nor a purely topological approach, but rather I use both algebraic and topological techniques as they seem appropriate. The first six chapters contain what I consider to be the basics of the subject. The remaining four chapters are somewhat more specialized and reflect my own research interests. For the most part, the only prerequisites for reading the book are the elements of algebra (groups, rings, and modules, including tensor products over non-commutative rings) and the elements of algebraic topology (fundamental group, covering spaces, simplicial and CW-complexes, and homology). There are, however, a few theorems, especially in the later chapters, whose proofs use slightly more topology (such as the Hurewicz theorem or Poincaré duality).

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书籍目录

Introduction CHAPTER Some Homological Algebra 0. Review of Chain Complexes 1. Free Resolutions
2. Group Rings 3. G -Modules 4. Resolutions of Z Over ZG via Topology 5. The Standard Resolution 6.
Periodic Resolutions via Free Actions on Spheres 7. Uniqueness of Resolutions 8. Projective Modules
Appendix. Review of Regular Coverings CHAPTER The Homology of a Group 1. Generalities 2.
Co-invariants 3. The Definition of $H_n(G)$ 4. Topological Interpretation 5. Hopf's Theorems 6. Functoriality
7. The Homology of Amalgamated Free Products Appendix. Trees and Amalgamations CHAPTER
Homology and Cohomology with Coefficients 0. Preliminaries on X/G and $\text{Hom}G$ 1. Definition of $H_n(G, M)$
and $H^*(G, M)$ 2. Tor and Ext 3. Extension and Co-extension of Scalars 4. Injective Modules 5. Induced and
Co-induced Modules 6. H_n and H^* as Functors of the Coefficient Module 7. Dimension Shifting 8. H_n and H^* as
Functors of Two Variables 9. The Transfer Map 10. Applications of the Transfer CHAPTER Low
Dimensional Cohomology and Group Extensions 1. Introduction 2. Split Extensions 3. The Classification of
Extensions with Abelian Kernel 4. Application: p -Groups with a Cyclic Subgroup of Index p 5. Crossed
Modules and H^3 (Sketch) 6. Extensions With Non-Abelian Kernel (Sketch) CHAPTER Products 1. The
Tensor Product of Resolutions 2. Cross-products 3. Cup and Cap Products 4. Composition Products 5.
The Pontryagin Product 6. Application: Calculation of the Homology of an Abelian Group CHAPTER
Cohomology Theory of Finite Groups 1. Introduction 2. Relative Homological Algebra 3. Complete
Resolutions 4. Definition of $H^*(G, M)$ 5. Properties of $H^*(G, M)$ 6. Composition Products 7. A Duality Theorem 8.
Cohomologically Trivial Modules 9. Groups with Periodic Cohomology CHAPTER Equivariant Homology
and Spectral Sequences 1. Introduction 2. The Spectral Sequence of a Filtered Complex CHAPTER
Finiteness Conditions CHAPTER Euler Characteristics CHAPTER Farrell Cohomology
Theory References Notation Index Index

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精彩短评

- 1、觉得有用就买了
- 2、经典,爬了一小部分.

1、对于有限群而言，除了通常意义上的同调与上同调之外，还有一类专门的Tate Cohomology，其指标可以在所有整数上有定义，可以视为同调与上同调合体后的产物。对于具有相当数学素养的人而言，这无疑是一件非常美妙的事情，下面我就来具体阐述一下。我们知道群的同调可以通过投射分解计算，上同调则可以通过内射分解计算，这里我们要把投射分解与内射分解结合起来，构造出一个两边都开放的完全分解，其上同调群就Tate Cohomology.一般状况下，投射模与内射模并没有明显的联系，这里我们需要减弱条件，进入到相对同调代数的领域，引入所谓相对内射模的概念。实际上，相对内射模就是说对G与其有限指标子群H，当单射 $i: M \rightarrow N$ 视为H-模可裂时，作为G-模的诱导映射 i^* 是满射。利用上诱导与限制的平衡性，可以证明对任何H-模N，上诱导G-模 $\text{Coind}(N)$ 是相对内射的，进而任何G-模M均可嵌入典型相对内射模 $\text{Coind}(\text{Res}(M))$ ，并且当M是(有限生成)投射模时，被嵌入的也是(有限生成)投射模。这样的嵌入关系就导致了链式反应，得到一个投射模的后项分解，进而得到了由投射模组成的完全分解。对于完全分解而言，借助于对偶的概念，还有更加简明的刻画。对交换环R，记 $F^* = \text{Hom}(F, R)$ 为F的对偶，它可以把普通投射分解变成后项的投射分解。可以先取两个投射分解P与Q，Q的对偶 Q^* 就与P组成了完全分解，它结构形如... $P_0 \rightarrow Z \rightarrow Q_0 \rightarrow \dots$ ，复合一下可得... $P_0 \rightarrow Q_0 \rightarrow \dots$ ，再把所有 Q_i 记为 P_{-i-1} ，便得到了由 $\{P_i: i \in \mathbb{Z}\}$ 构成的完全分解。这样构造的Tate Cohomology借助了原先的分解，与普通上同调的差别仅在于“接缝”的地方。具体来说就是：当 $i > 0$ 时， $\text{Tate } H^i = H^i$ ；当 $i < -1$ 时， $\text{Tate } H^i = H^{-(i+1)}$ 。而 $\text{Tate } H^{-1} = \ker N$ ， $\text{Tate } H^0 = \text{coker } N$ ，这里N称为范映射，实际上就是G内所有元素之和(G有限!)。下面我们看一下循环群的例子，对于循环群 Z_p ，普通上同调的情况是： $H^i(Z_p) = Z_p$ ，若p是正偶数； $H^i(Z_p) = Z$ ，若p=0； $H^i(Z_p) = 0$ ，若p是奇数或负的。而相应的Tate Cohomology则是： $\text{Tate } H^i(Z_p) = Z_p$ ，若p是偶数 $\text{Tate } H^i(Z_p) = 0$ ，若p是奇数显然，Tate Cohomology的结论消除了零阶这个“奇点”，无疑要比普通上同调完善很多，事实上我们有结论群G的Tate Cohomology是周期为2的iff G是循环群。除了周期为2的情况之外，一般情况又如何呢？实际上，像同调长正合列、函子可消性、杯积对偶等等的性质，Tate Cohomology都与普通上同调类似，但这个周期上同调却是Tate Cohomology特有的有趣性质。对于周期上同调，一个简明的等价条件是：G有周期上同调iff对某 $d \neq 0$ ， $\text{Tate } H^d(G, Z) \cong Z/|G|Z(\cong \text{Tate } H^0(G, Z))$ 实际上就是说可以把它“平移”到原点处理，具体可以解释为存在元素 $u \in \text{Tate } H^d(G, Z)$ 可逆，其同构可以由 $-u$ 诱导。由此我们得到一个简单的推论：若G有周期上同调，则G的任何子群H也是如此。接下来研究带有周期上同调的群有何性质，这可以通过p-群进行处理：有限群G有周期上同调iff G的任何Sylow子群有周期上同调。我们先研究p-群G的周期上同调，假若G包含形如 $Z_p \times Z_p$ 的子群，那么由Kunneth公式可得 $\text{Tate } H^n(Z_p \times Z_p, Z_p)$ 作为 Z_p -向量空间是 $n+1$ 维的，因此它就是不是周期的。这样我们得到，若p-群G有周期上同调，则p的任何Abel子群都是循环的，进而得到G有唯一的p阶子群，再引用一下群论中的定理，最终得到G是循环群或广义四元数群(此时 $p=2$)。幸运的是，这两种群确实有周期上同调，这就完成了整个循环的推理。把p-群的结论应用到一般情况，可以得到结论：G有周期上同调iff G的任何Sylow子群是循环群或广义四元数群。此外，我们还可以轻装上阵，只做局部化处理。考虑对某个固定素数p的周期上同调，便可得到结论：G有p-周期上同调iff G的p-Sylow子群是循环群或广义四元数群。最后简单提一下Tate Cohomology的推广，实际上它可以推广到满足一定有限条件的(无限)群上。具体来说，就推广到 $\text{vcd}(G)$ 有限的群上，即存在某个有限指标的子群H，满足其上同调维数 $\text{cd}(H)$ 有限。对于这样的群，我们也可以构造某种完全分解，所得到的上同调称为Farrell Cohomology，对此本文就不再详细阐述了。原文地址

: http://blog.sina.com.cn/s/blog_486c2cbf0102e0rt.html

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